

Structural Analysis 1

HW # 7

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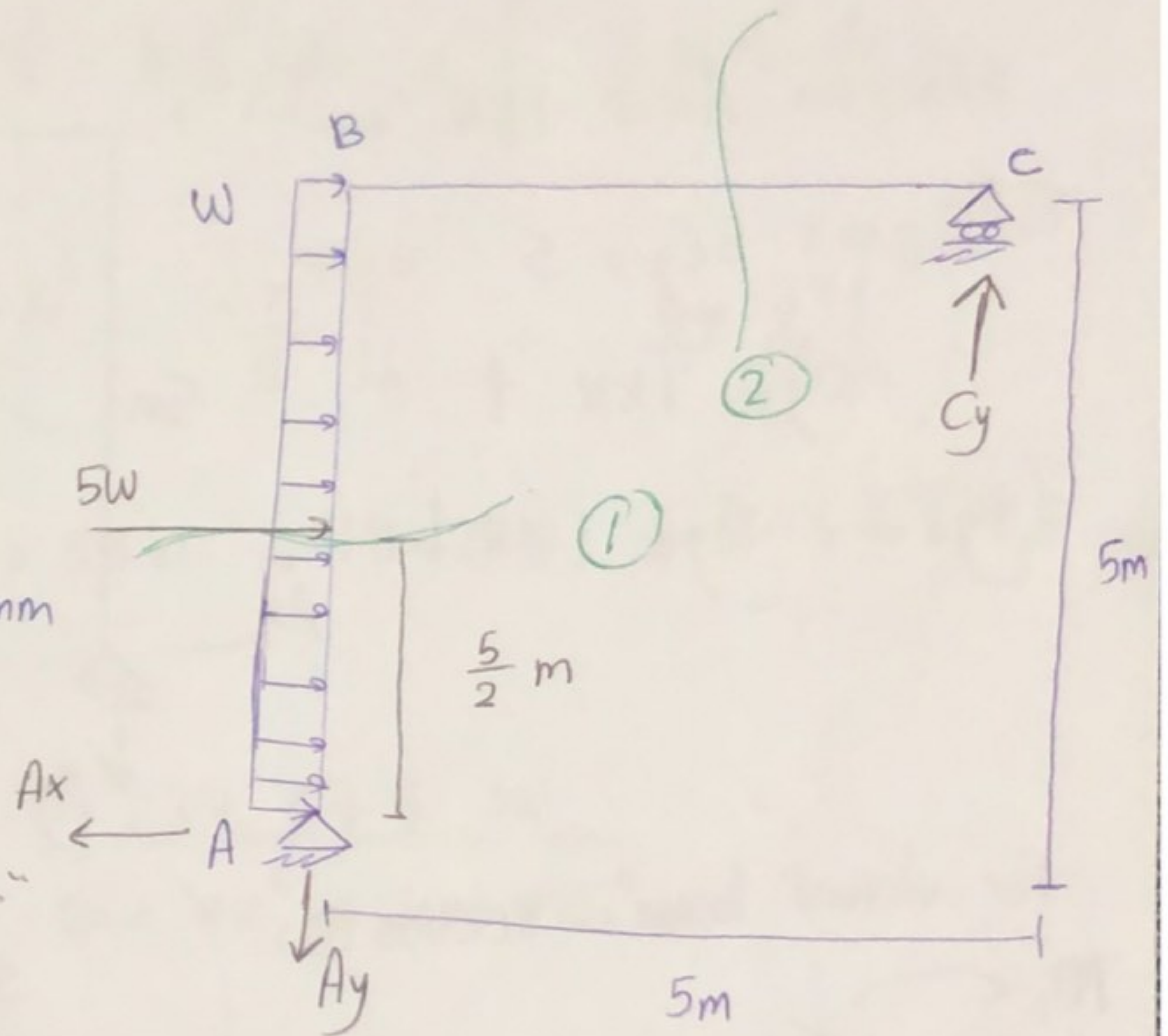
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Q1: - (1)

$$E = 200 \text{ GPa}$$

$$I = 2000 \times 10^6 \text{ mm}^4$$

find W where $\Delta_c^{\text{max}} = 9 \text{ mm}$



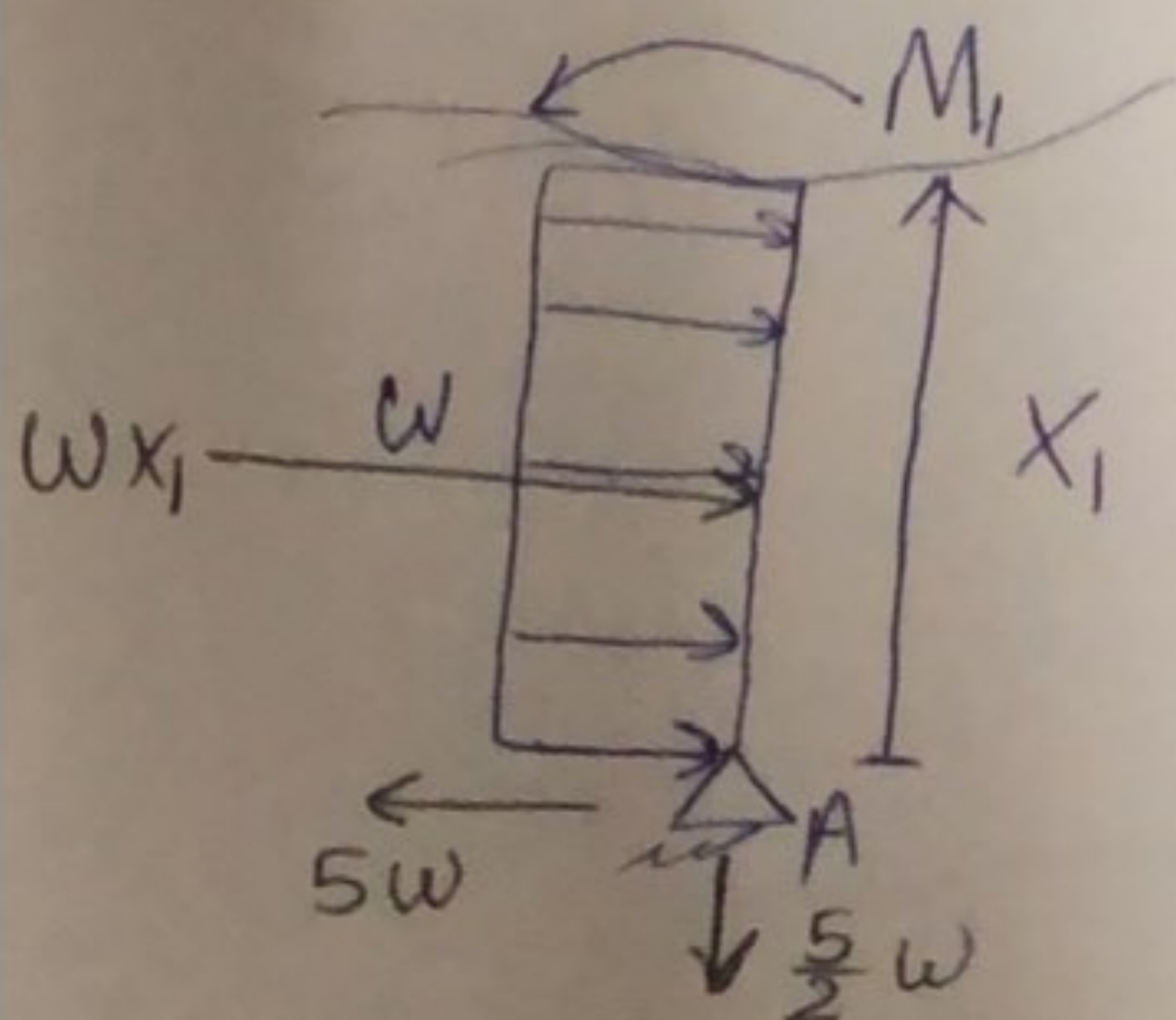
$$\sum F_x = 0: A_x = 5W \leftarrow$$

$$\sum M_A = 0: 5C_y = \frac{25}{2} W$$

$$C_y = \frac{5}{2} W \uparrow$$

$$\sum F_y = 0: A_y = \frac{5}{2} W \downarrow$$

for real Beam: section (1):

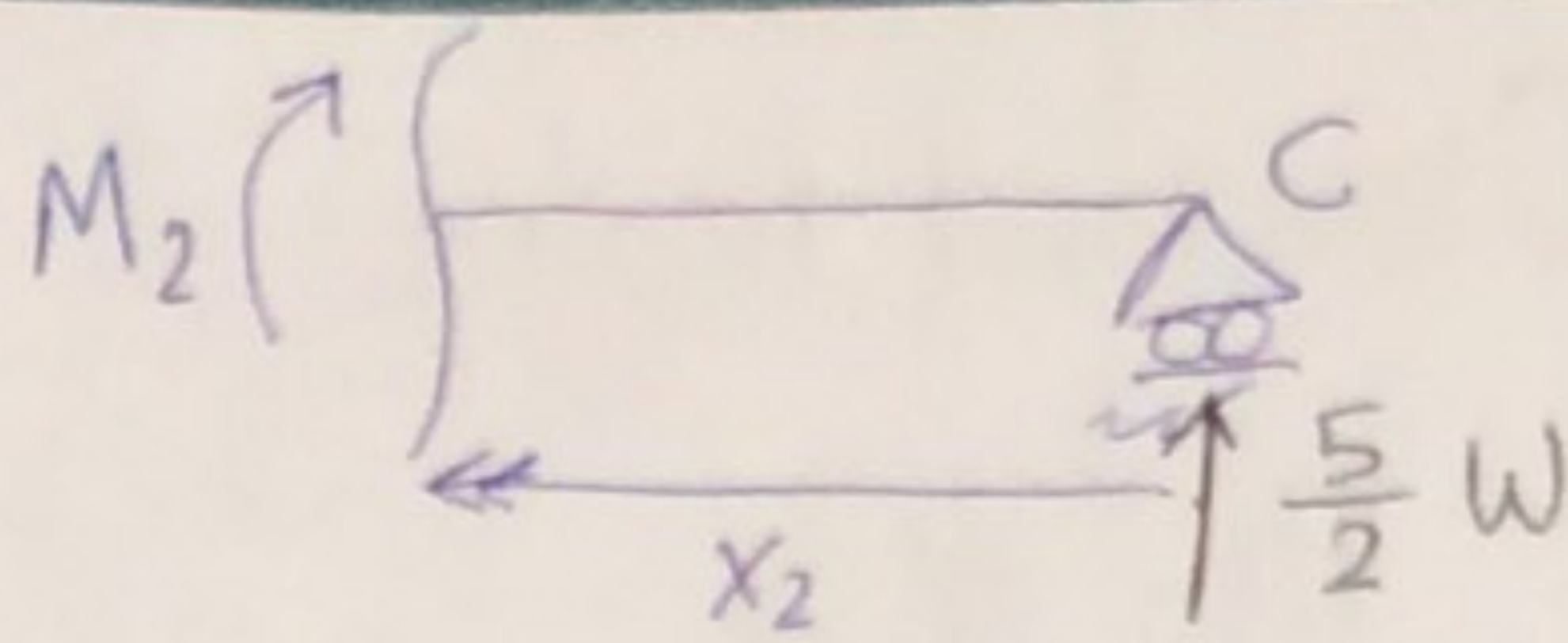


$$M_i = 5W x_1 - \frac{W x_1^2}{2} \quad (x_1: 0 \rightarrow 5)$$

$$0 < x_1 < 5$$

1

section ②:



$$M_2 = \frac{5}{2} W x_2 \quad " 0 < x_2 < 5 "$$

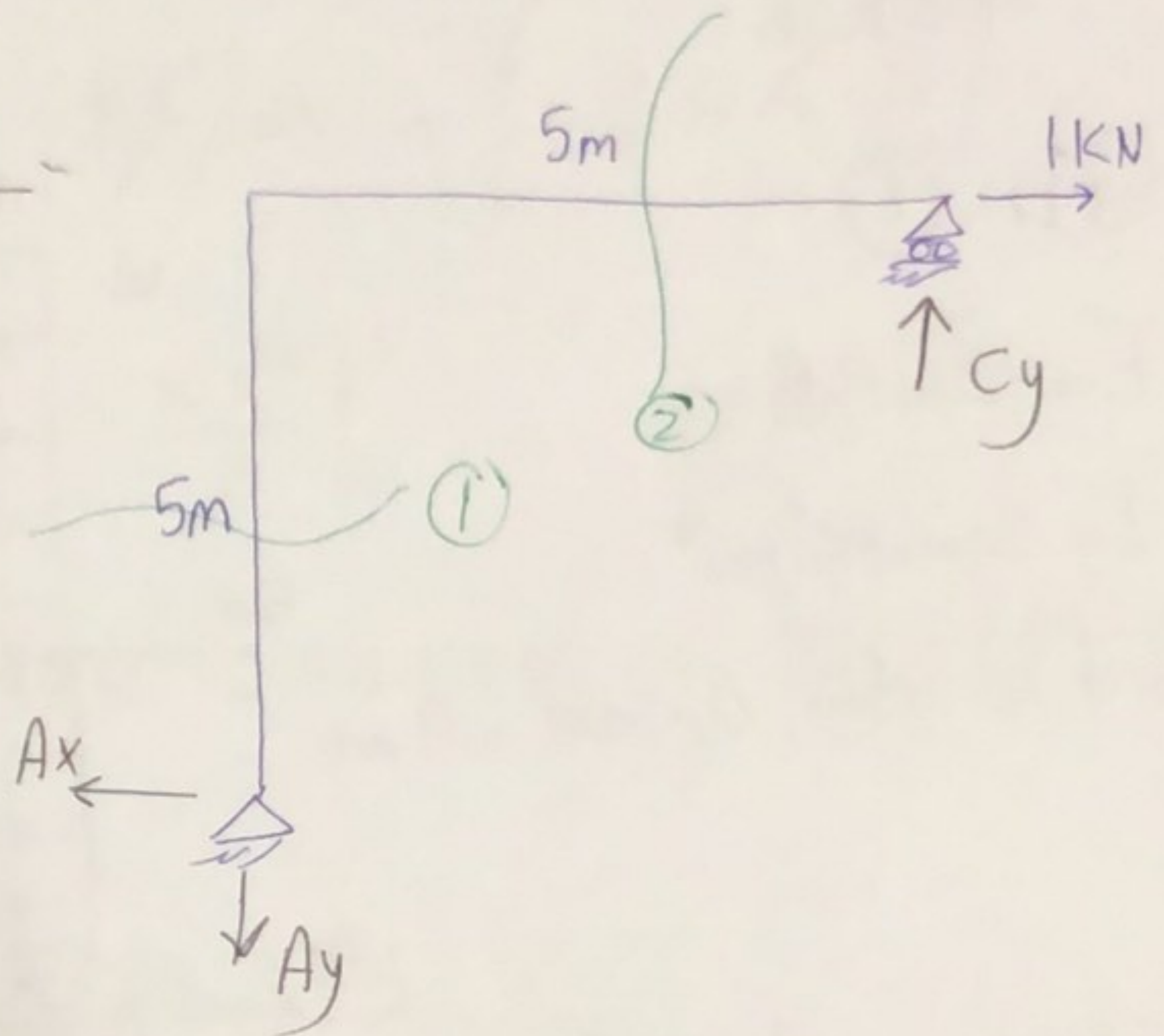
Virtual ~~displacement~~ :

$$\Sigma F_x = 0 : A_x = 1 \text{ KN } \leftarrow$$

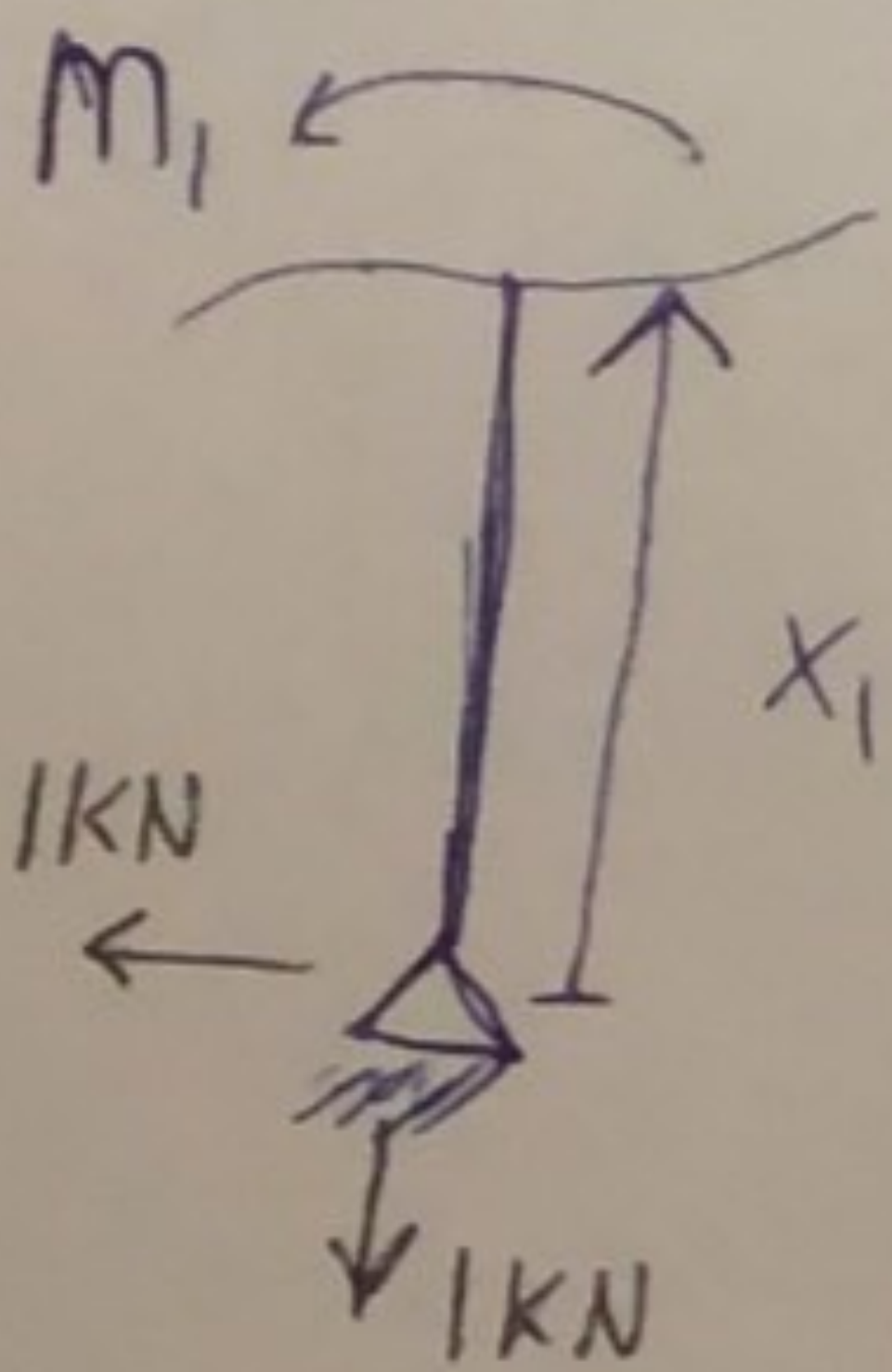
$$\Sigma M_A = 0 : 5 C_y = 5$$

$$C_y = 1 \text{ KN } \uparrow$$

$$\Sigma F_y = 0 : A_y = 1 \text{ KN } \downarrow$$



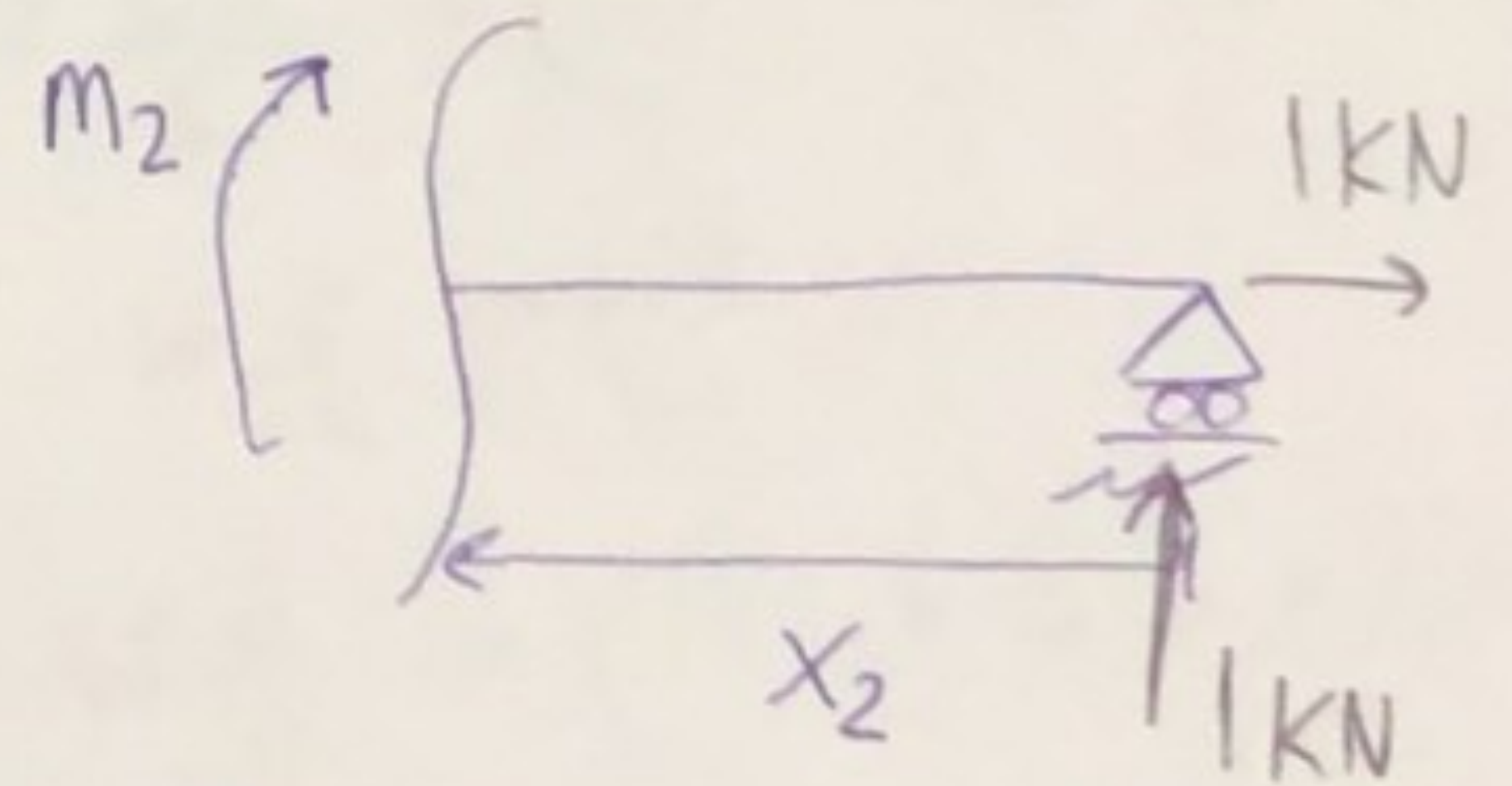
For virtual ~~displacement~~ : section ①:



$$M_1 = x_1$$

$$" 0 < x_1 < 5 "$$

section ②:



$$M_2 = x_2 \quad " 0 < x_2 < 5 "$$

$$I * \Delta_c = \int_0^5 \frac{M_1 m_1}{EI} dx + \int_0^5 \frac{M_2 m_2}{EI} dx$$

$$\Delta_c = \frac{I}{EI} \left(\int_0^5 \left(5w x_1 - \frac{w x_1^2}{2} \right) x_1 dx + \int_0^5 \frac{5}{2} w x_2 \cdot x_2 dx \right)$$

$$\rightarrow \Delta_c = \frac{I}{EI} \left(\int_0^5 \left(5w x_1^2 - \frac{w x_1^3}{2} \right) dx + \int_0^5 \frac{5}{2} w x_2^2 dx \right)$$

$$\rightarrow \Delta_c = \frac{I}{EI} \left(\frac{5w}{3} x_1^3 - \frac{w x_1^4}{8} \Big|_0^5 + \frac{5w}{6} x_2^3 \Big|_0^5 \right)$$

$$\rightarrow \Delta_c = \frac{I}{EI} \left(208.333w - 78.125w + 104.167w \right)$$

$$0.009 \text{ m} = \frac{234.375 w}{200 * 10^6 * 2000 * 10^{-6}}$$

$$\rightarrow w = 15.36 \text{ KN/m}$$

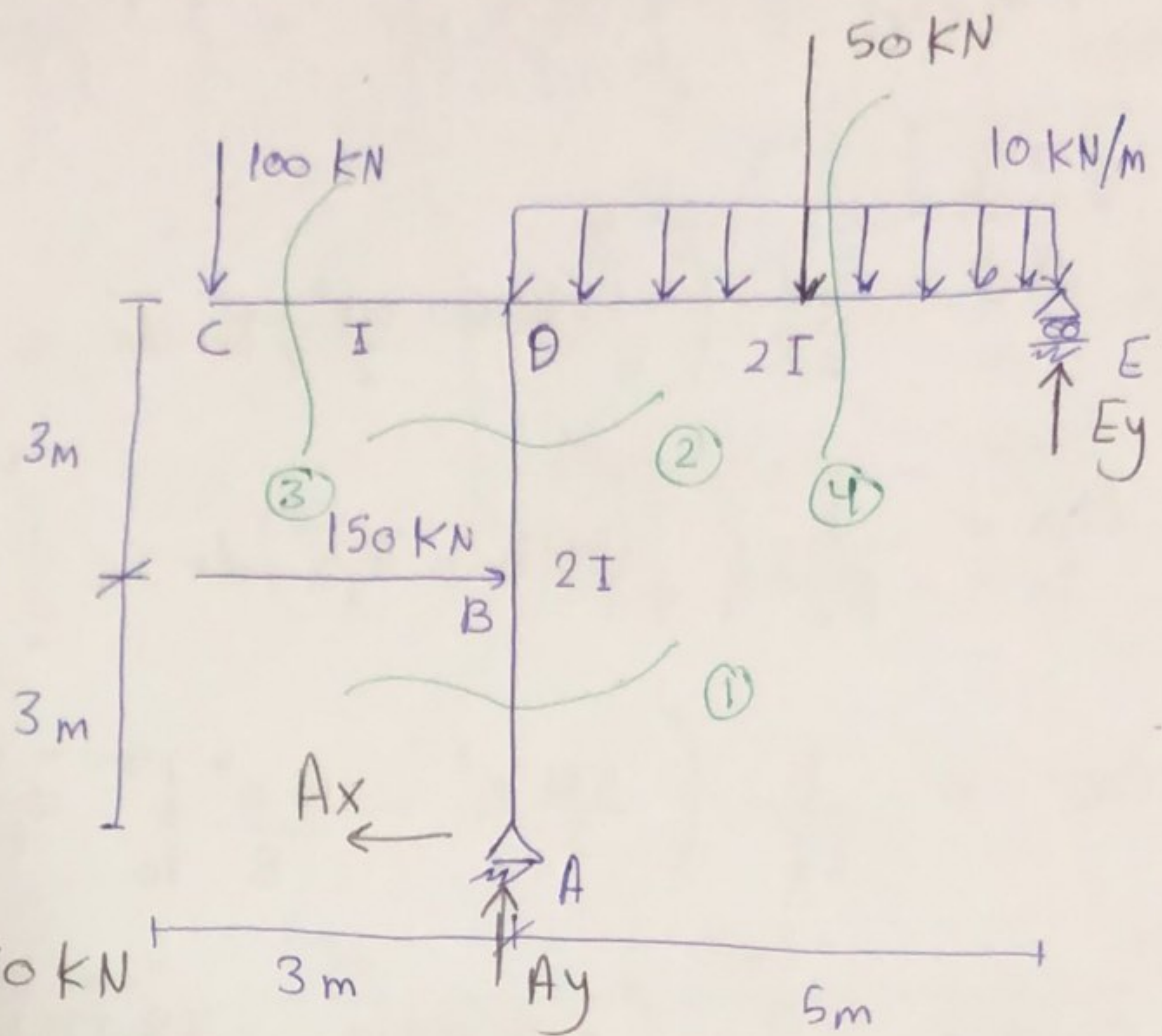
Q1: ②

a) Δ_E "horizontal"

b) Δ_C "vertical"

$E = 200 \text{ GPa}$

$I = 500 \times 10^6 \text{ mm}^4$



$\Sigma F_x = 0: A_x = 150 \text{ kN}$

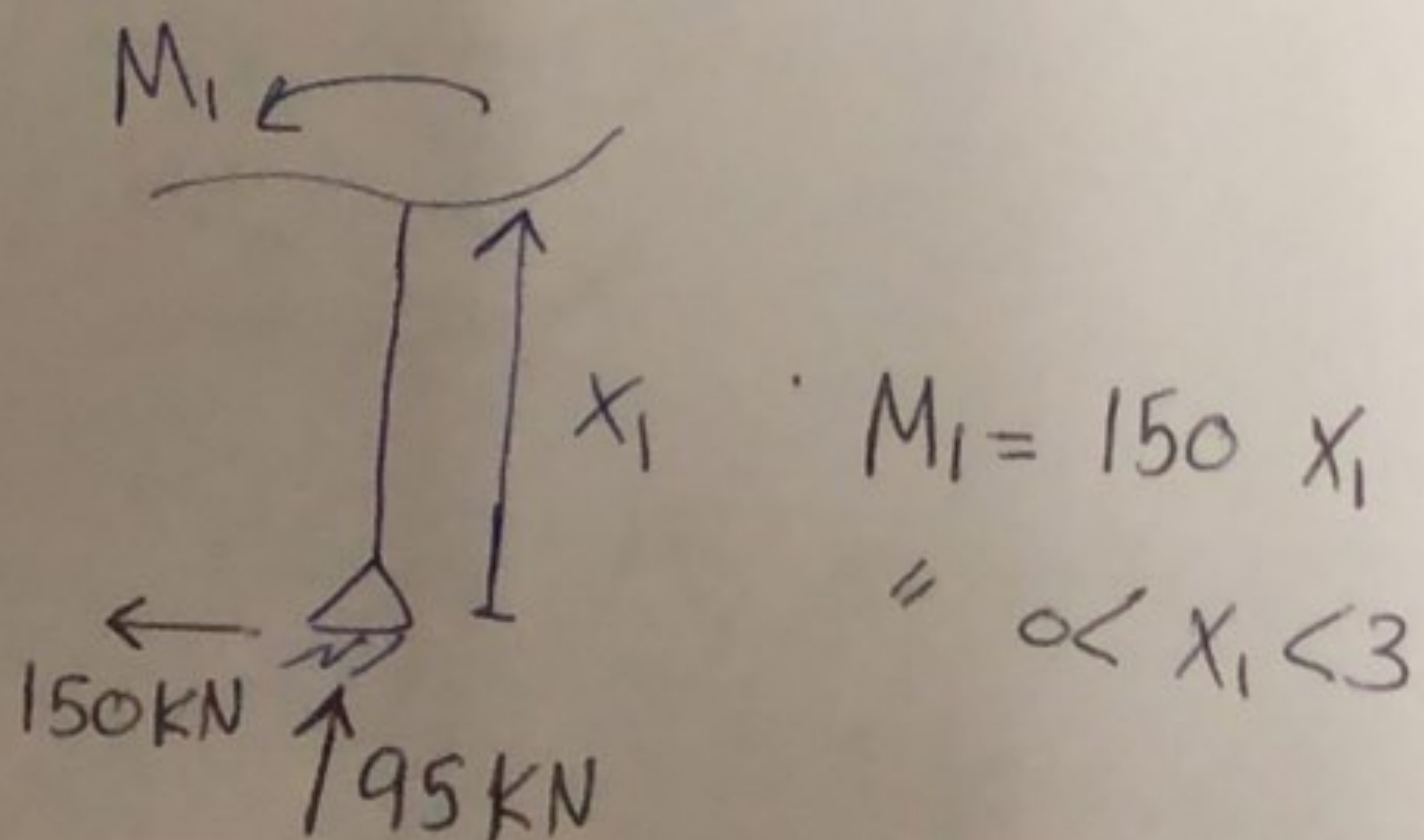
$\Sigma M_A = 0: 5 E_y = 125 + 450 - 300$

$E_y = 55 \text{ kN} \uparrow$

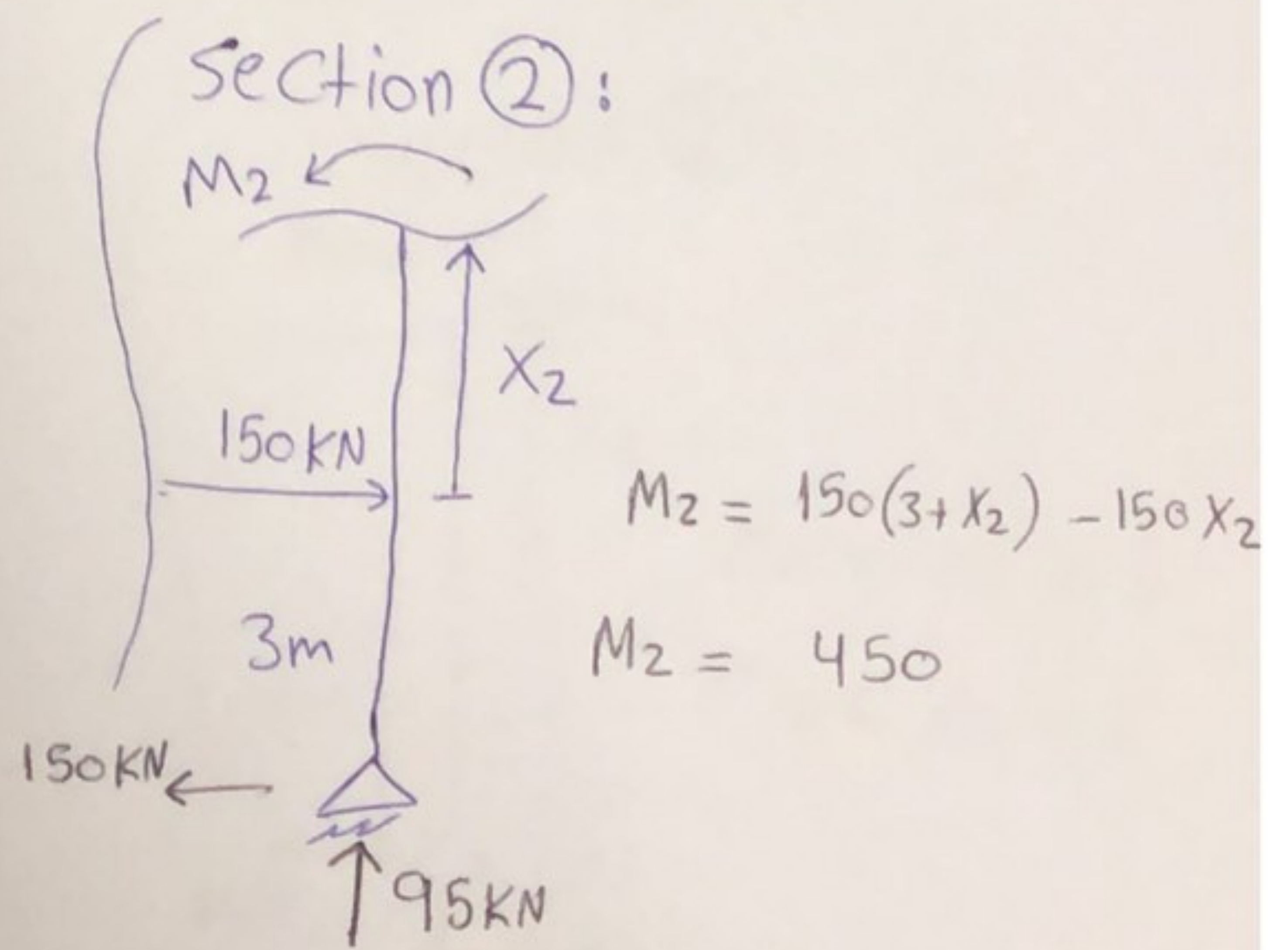
$\Sigma F_y = 0: A_y = 95 \text{ kN} \uparrow$

For Real Beam:

section ①:



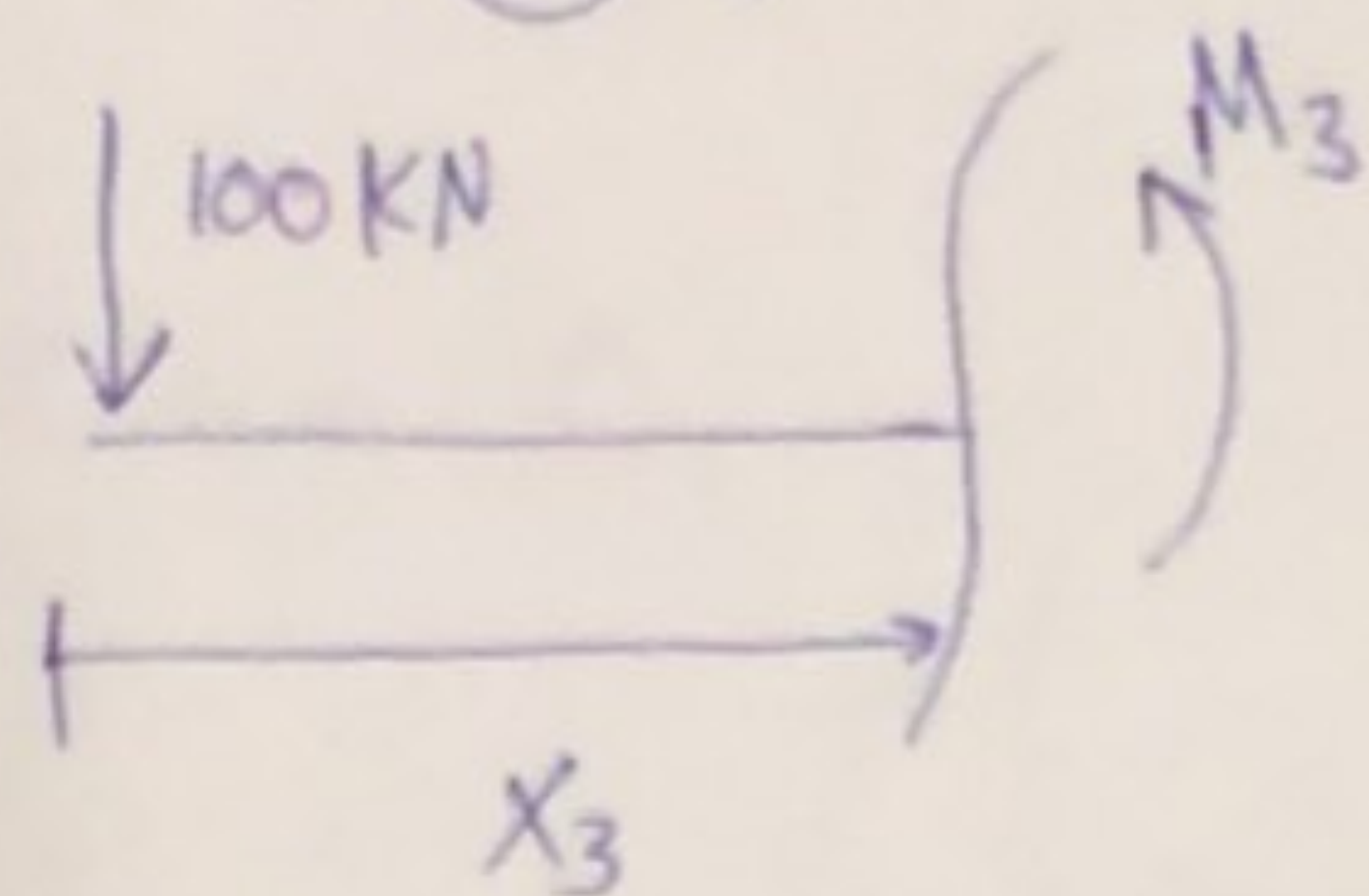
section ②:



$(0 < x_2 < 3)$

4

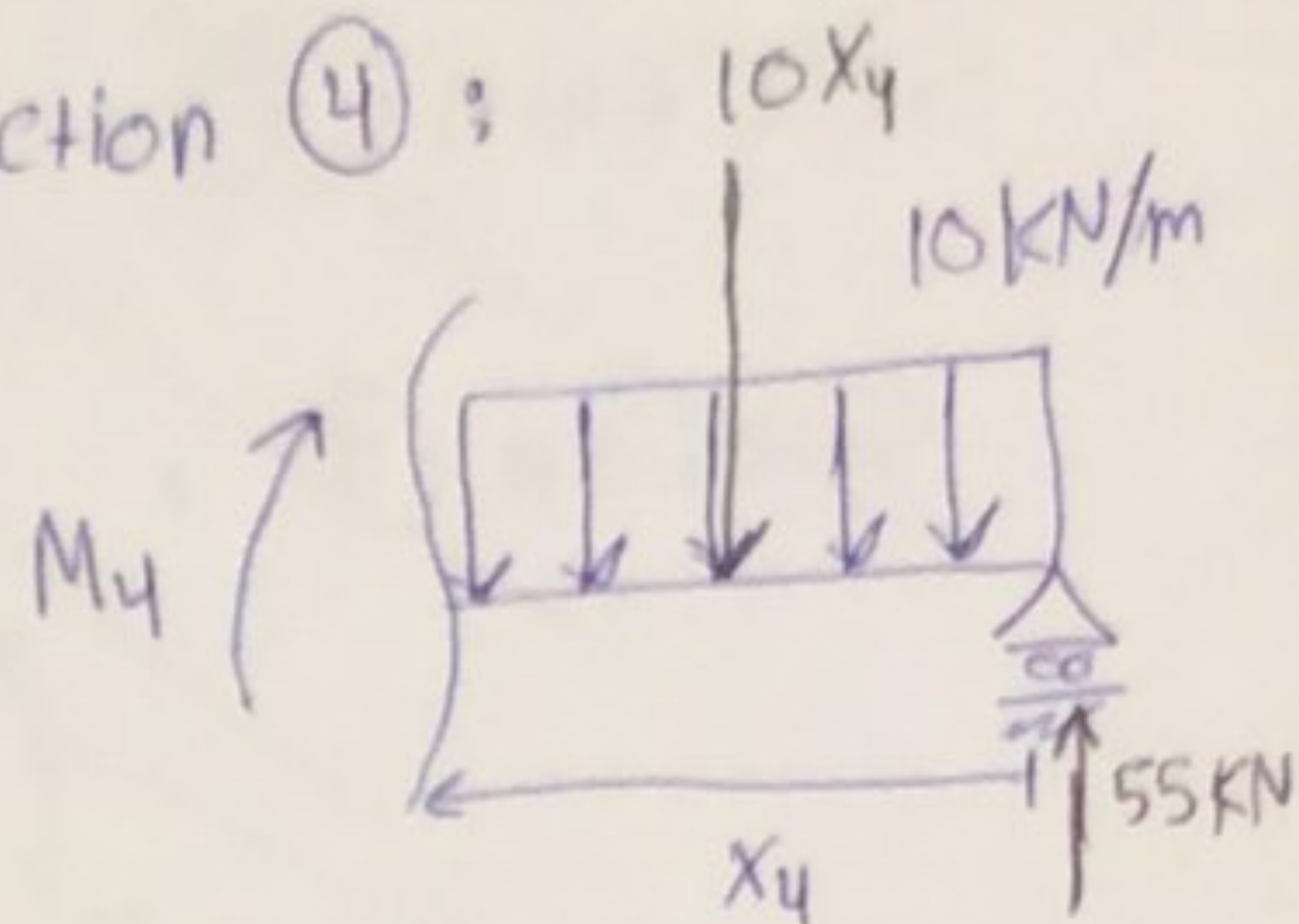
Section ③:



$$M_3 = -100X_3$$

$$0 < X_3 < 3$$

Section ④:



$$M_4 = 55X_4 - \frac{10X_4^2}{2}$$

$$\rightarrow M_4 = 55X_4 - 5X_4^2$$

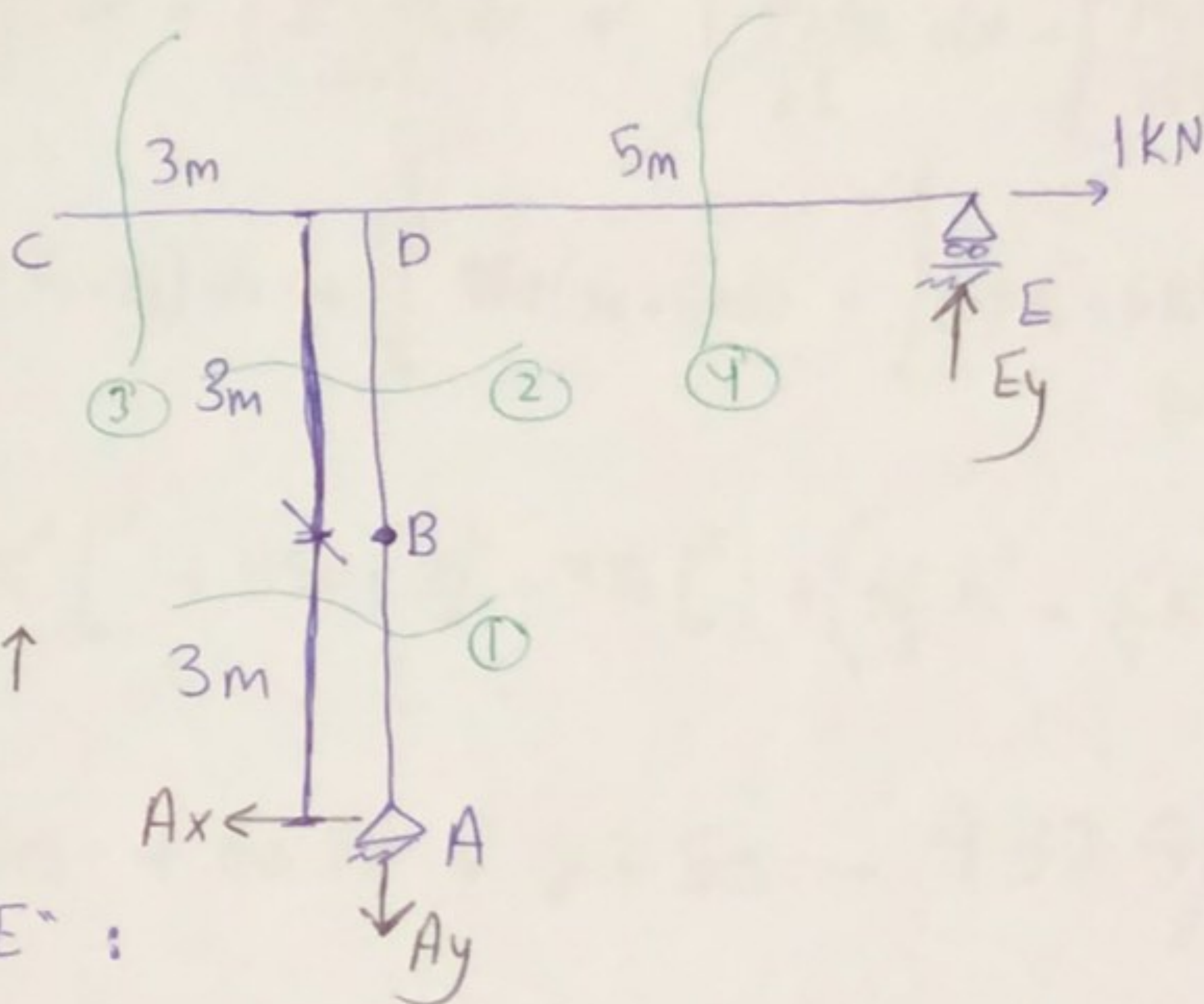
Virtual ~~Beam~~ "E":

$$\sum F_x = 0: A_x = 1 \text{ kN} \leftarrow$$

$$\sum M_A = 0: 5E_y = 6$$

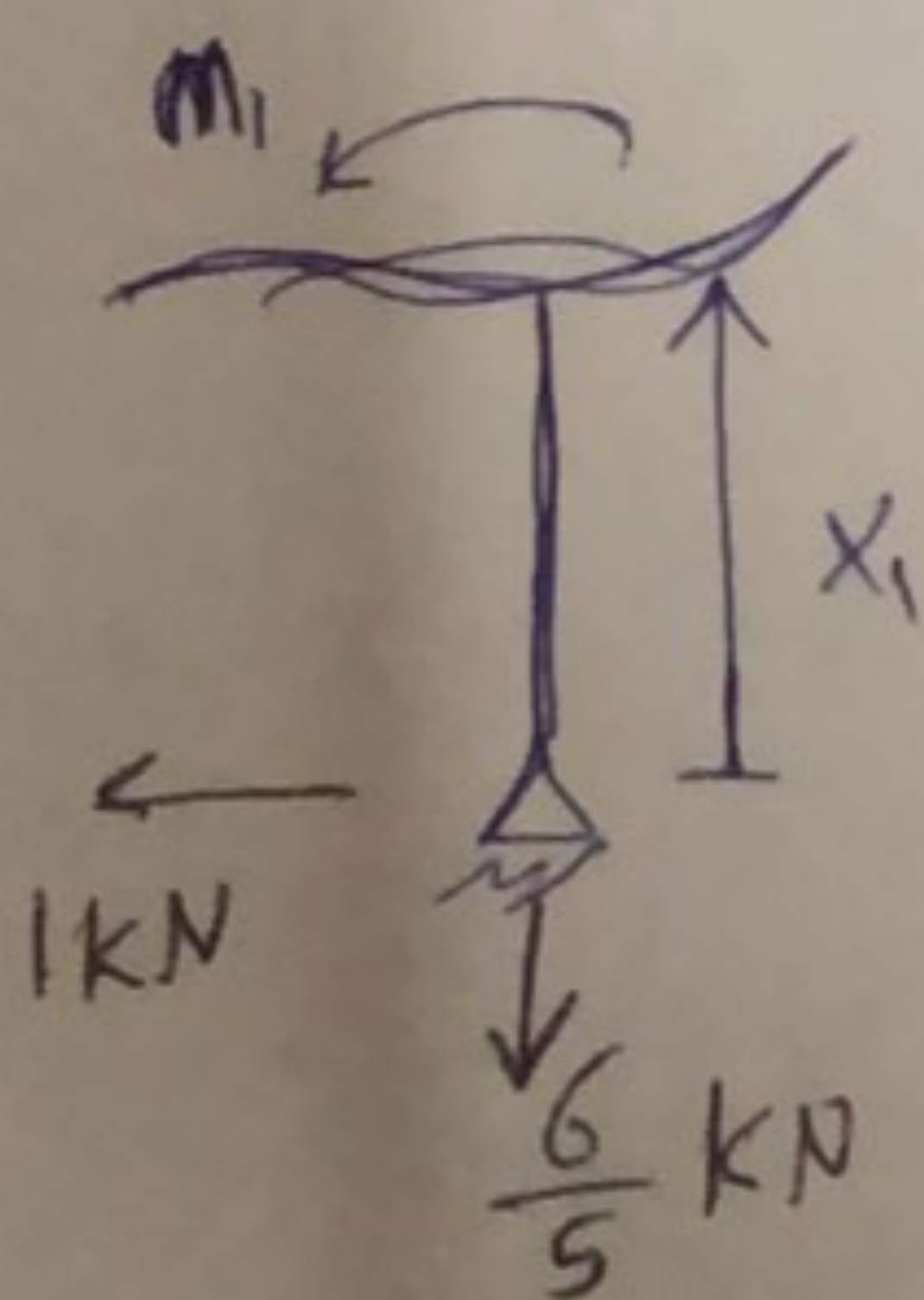
$$E_y = \frac{6}{5} \text{ kN} \uparrow$$

$$\sum F_y = 0: A_y = \frac{6}{5} \text{ kN} \uparrow$$



For virtual ~~Beam~~ "E":

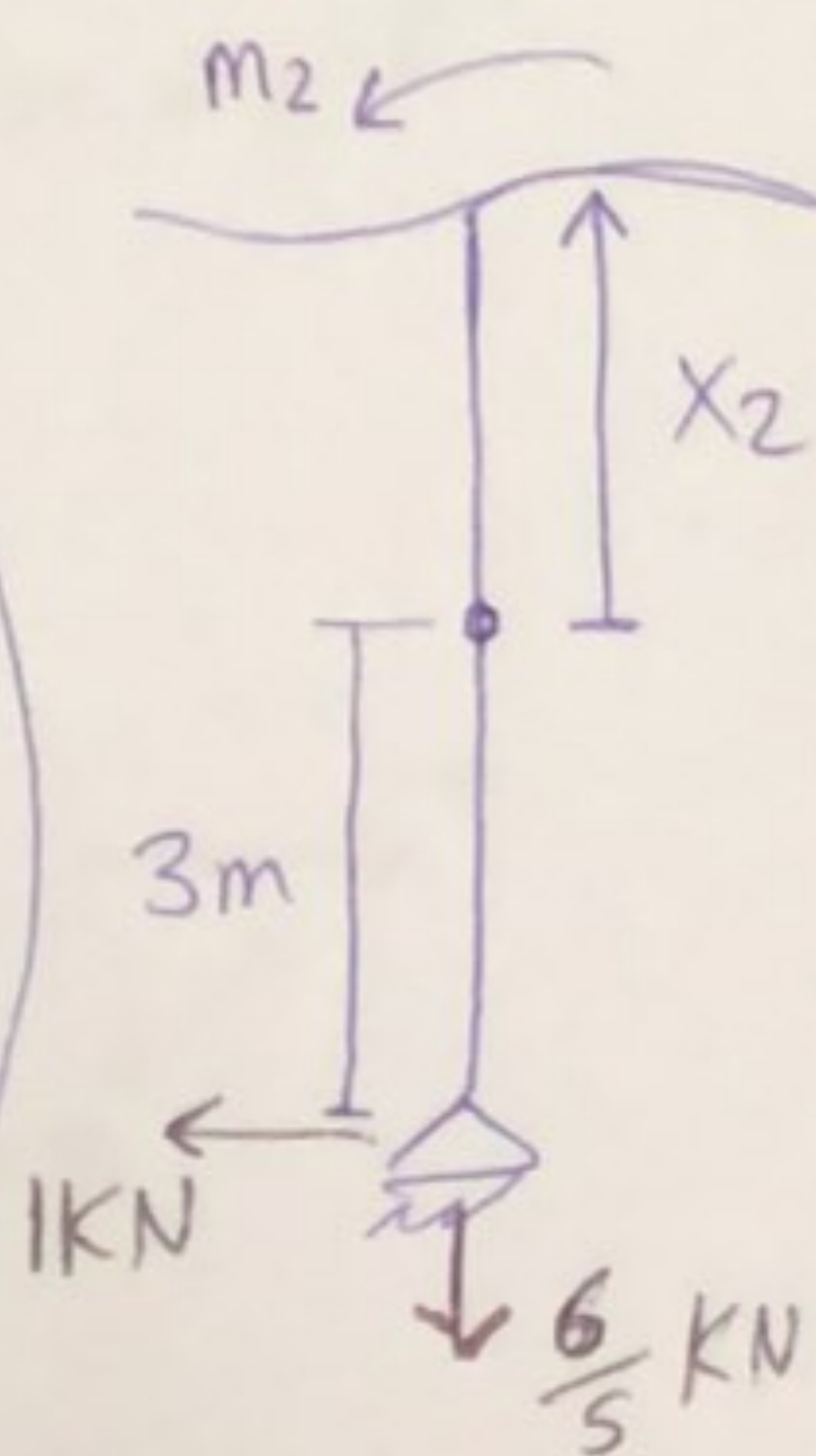
Section ①:



$$M_1 = X_1$$

$$0 < X_1 < 3$$

Section ②:

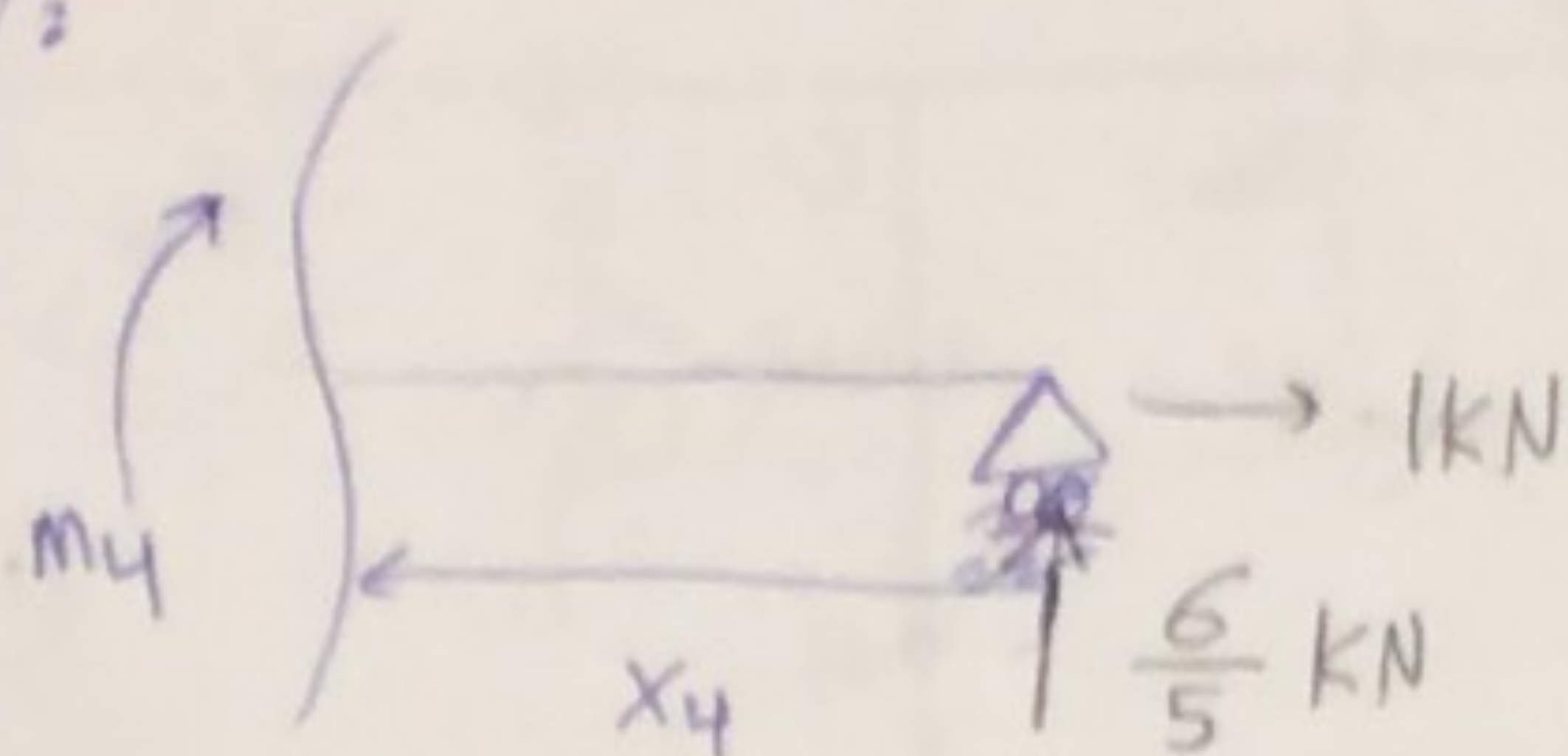


$$M_2 = X_2 + 3$$

$$0 < X_2 < 3$$

Section ③ : $m_3 = 0$

Section ④ :



$$m_4 = \frac{6}{5} x_4$$

$$1 * \Delta_E = \left(\int_0^3 \frac{M_1 m_1}{2EI} dx + \int_0^3 \frac{M_2 m_2}{2EI} dx + \int_0^3 \frac{M_3 m_3}{EI} dx + \int_0^5 \frac{M_4 m_4}{2EI} dx \right)$$

$$= \frac{1}{2EI} \left(\int_0^3 (150 x_1 \cdot x_1) dx + \int_0^3 450 (x_2 + 3) dx + \int_0^5 (55 x_4 - 5 x_4^2) \cdot \frac{6}{5} x_4 dx \right)$$

$$= \frac{1}{2EI} \left(\frac{150}{3} x_1^3 \Big|_0^3 + 450 \left(\frac{x_2^2}{2} + 3x_2 \Big|_0^3 \right) + \left(\frac{66}{3} x_4^3 - \frac{6}{4} x_4^4 \Big|_0^5 \right) \right)$$

$$= \frac{1}{2EI} (1350 + 6075 + 2750 - 937.5)$$

$$= \frac{9237.5}{2 * 200 * 10^6 * 500 * 10^{-6}}$$

$$\rightarrow \Delta_E = 0.0462 \text{ m}$$

$$= 46.2 \text{ mm " } \rightarrow \text{"}$$

direction ↙

6

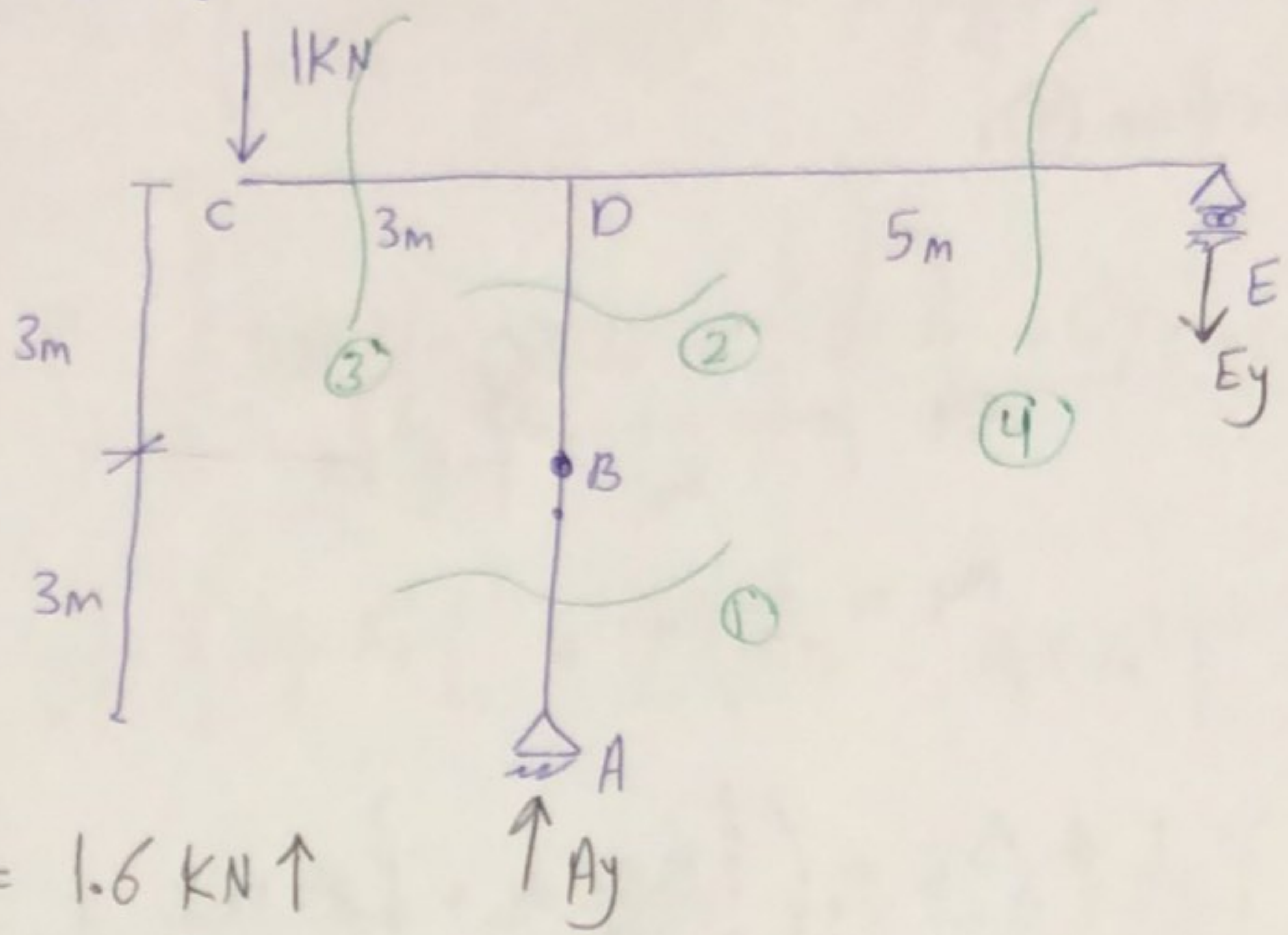
* Virtual ~~Reaction~~ "C" :-
for

$$\sum M_A = 0 :$$

$$3 = 5 E_y$$

$$E_y = 0.6 \text{ KN } \downarrow$$

$$\sum F_y = 0 : A_y = 1.6 \text{ KN } \uparrow$$



for virtual ~~Reaction~~ "C" :-

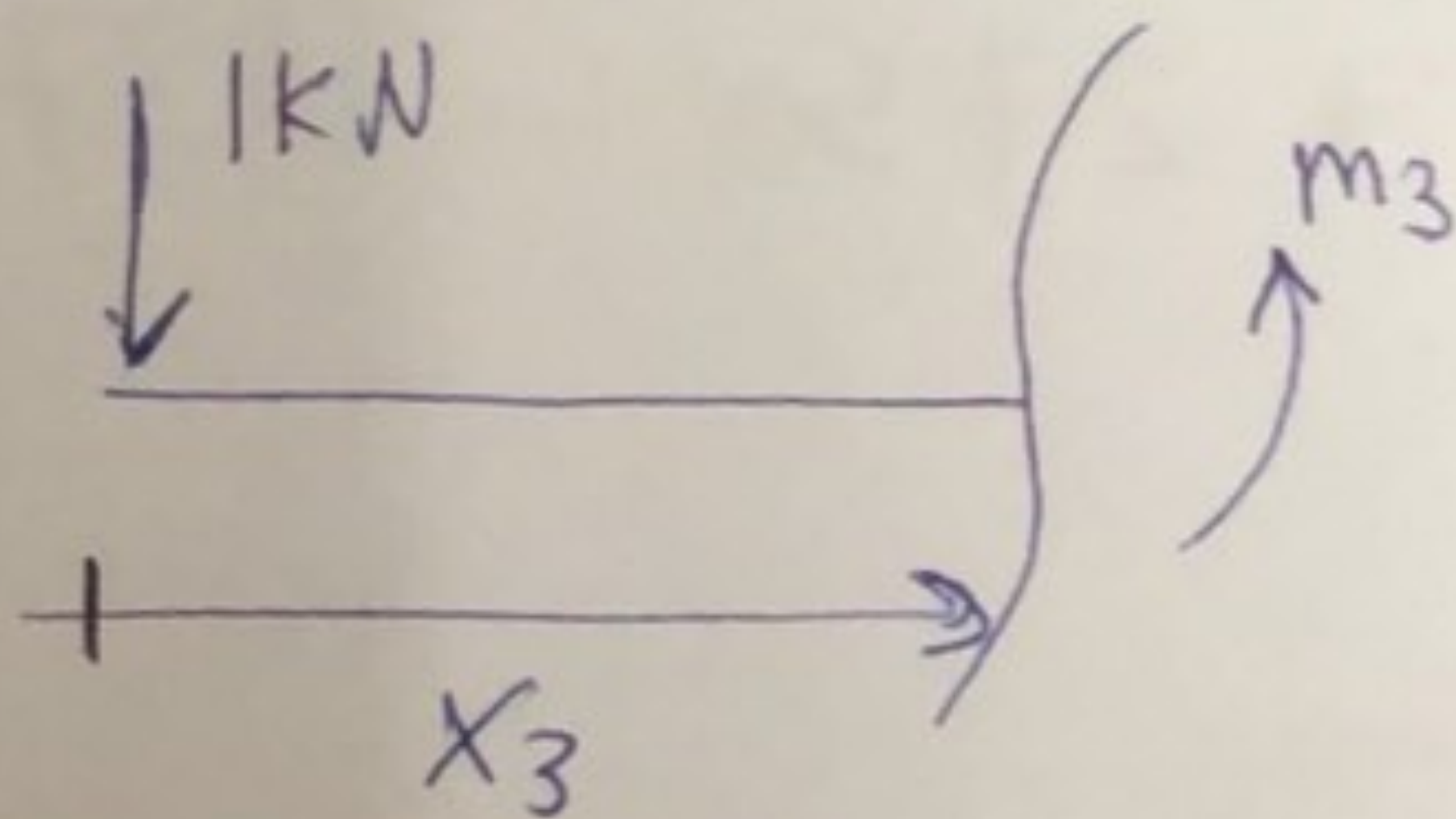
section (1) : $m_1 = 0$

~~section (2)~~

section (2) : $m_2 = 0$

no shear force in those beams

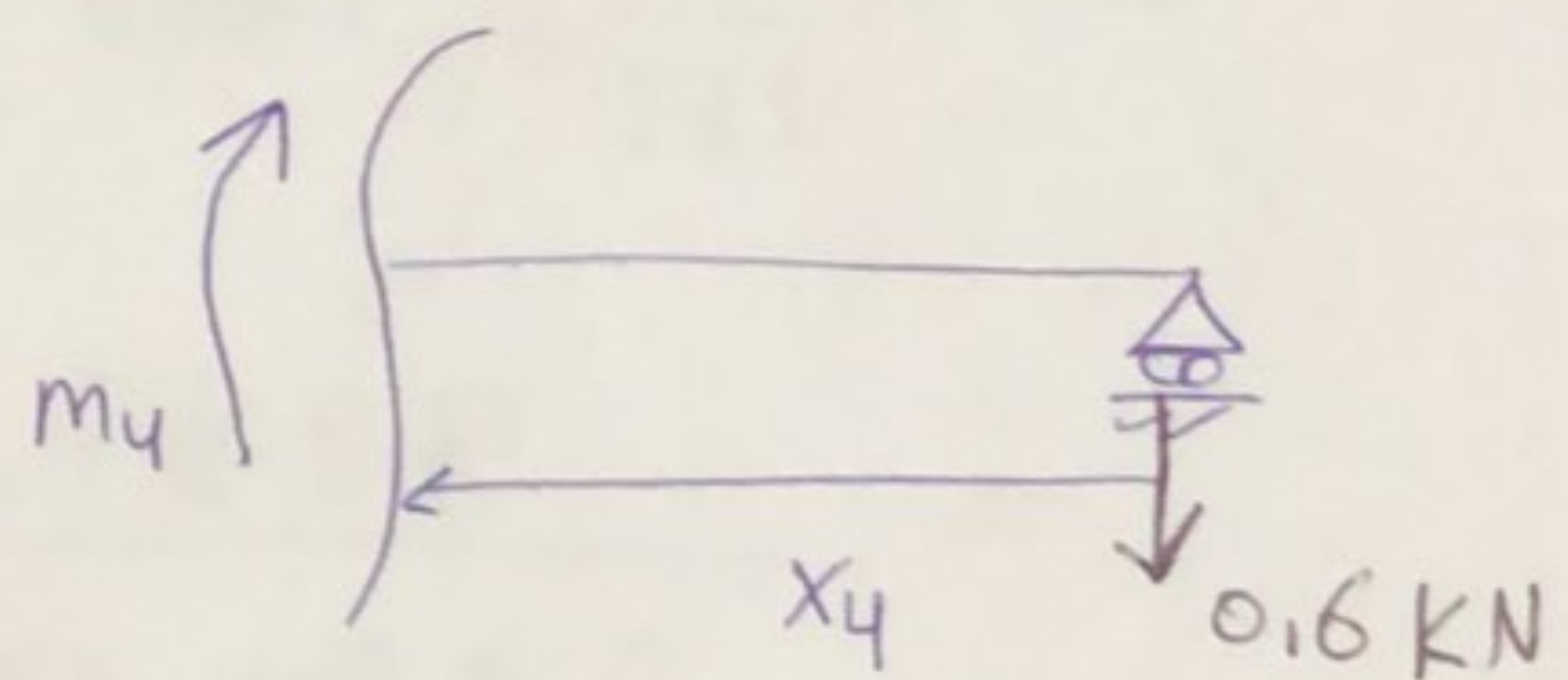
section (3) :



$$m_3 = -x_3$$

$$0 < x_3 < 3$$

section (4) :



$$m_4 = -0.6 x_4$$

$$0 < x_4 < 5$$

$$1 * \Delta_C = \left(\int_0^3 \frac{M_1 m_1}{2EI} dx + \int_0^3 \frac{M_2 m_2}{2EI} dx + \int_0^3 \frac{M_3 m_3}{EI} dx + \int_0^5 \frac{M_4 m_4}{2EI} dx \right)$$

$$\rightarrow \Delta_C = \int_0^3 \frac{-100 X_3 \cdot -X_3}{EI} dx + \int_0^5 \frac{(55 X_4 - 5 X_4^2) (-0.6 X_4)}{2EI} dx$$

$$= \frac{1}{EI} \left(\frac{100}{3} X_3^3 \Big|_0^3 + \left[\frac{1.5}{4} X_4^4 - \frac{16.5}{3} X_4^3 \Big|_0^5 \right] \right)$$

$$= \frac{1}{EI} (900 + 234.375 - 687.5)$$

$$\rightarrow \Delta_C = \frac{446.875}{200 * 10^6 * 500 * 10^{-6}}$$

$$\rightarrow \Delta_C = 0.00447 \text{ m } \downarrow$$

$$= 4.47 \text{ mm } \downarrow$$

direction

8

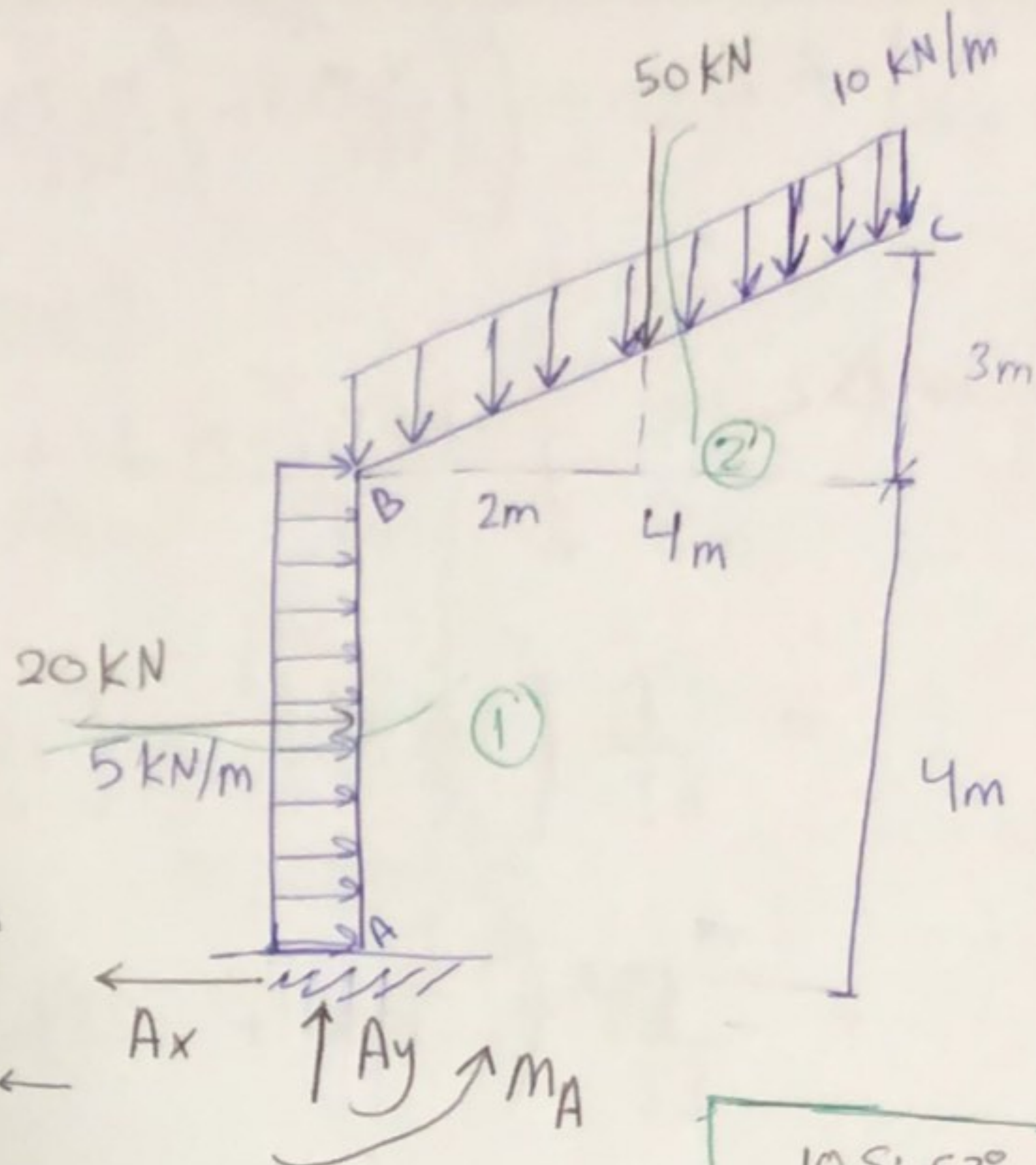
Q2 :-)

$\Delta_c ?$

$E = 25 \text{ GPa}$

($B = 25 \text{ cm} / H = 40 \text{ cm}$)

→ Cross Section



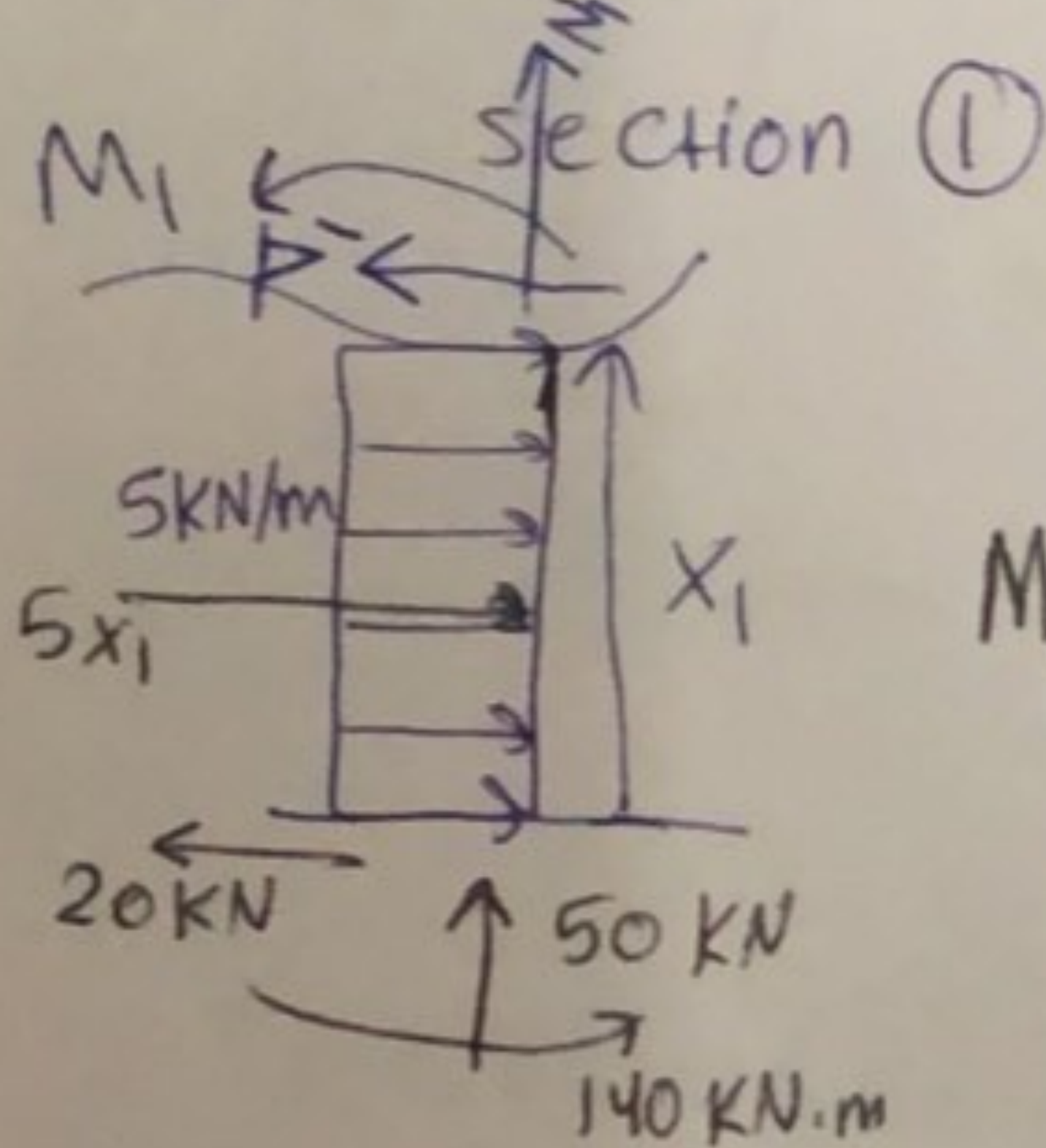
$\sum F_y = 0 : A_y = 50 \text{ kN} \uparrow$

$\sum F_x = 0 : A_x = 20 \text{ kN} \leftarrow$

$\sum M_A = 0 : M_A = 40 + 100 = 140 \text{ kN.m}$

$10 \sin 53^\circ = 8$
 $10 \cos 53^\circ = 6$

for Real :



$M_1 = 20x_1 - \frac{5x_1^2}{2} - 140$

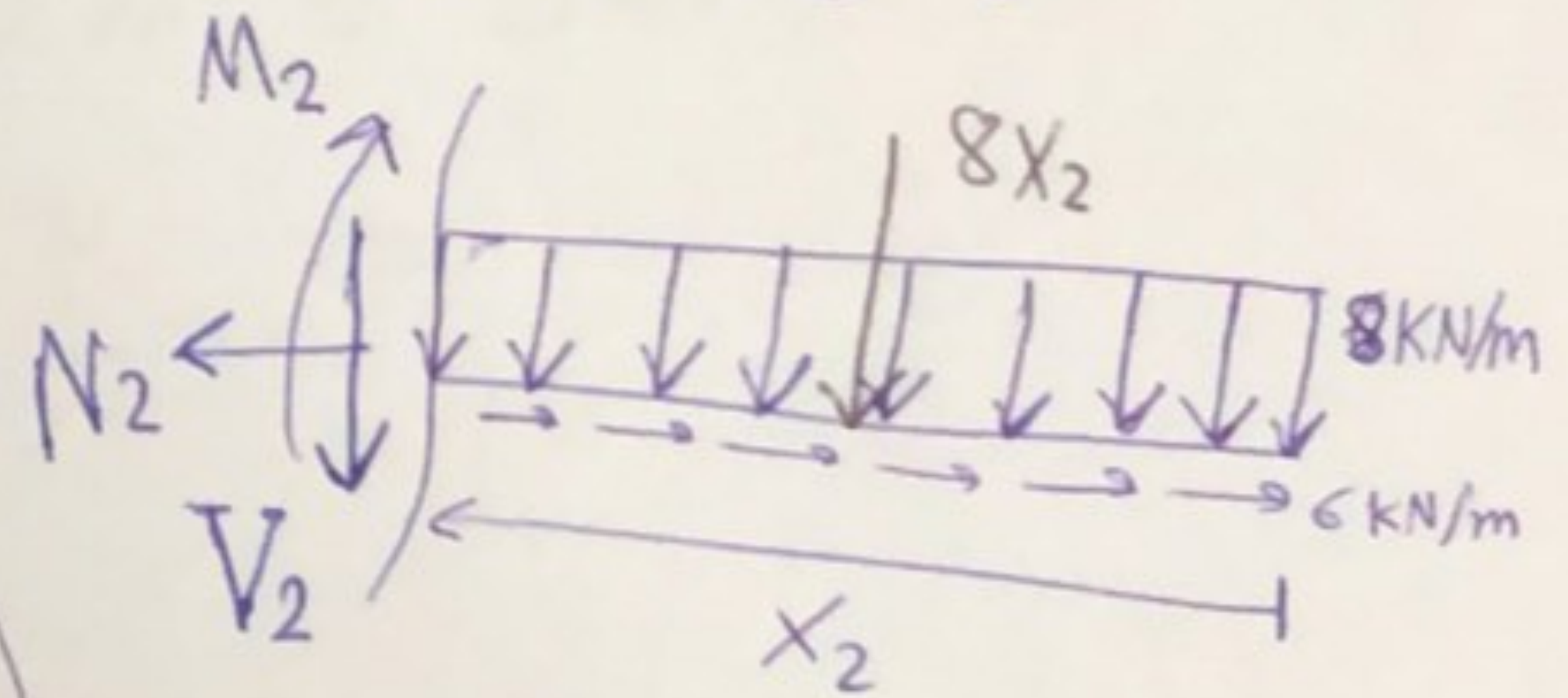
$M_1 = 20x_1 - \frac{5}{2}x_1^2 - 140$

$(0 < x_1 < 4)$

$N_1 = -50 \text{ kN}$

$V_1 = 5x_1 - 20$

Section 2 :



$M_2 = -\frac{8x_2^2}{2}$

$= -4x_2^2$

$(0 < x_2 < 5)$

$N_2 = 6x_2$

$V_2 = -8x_2$

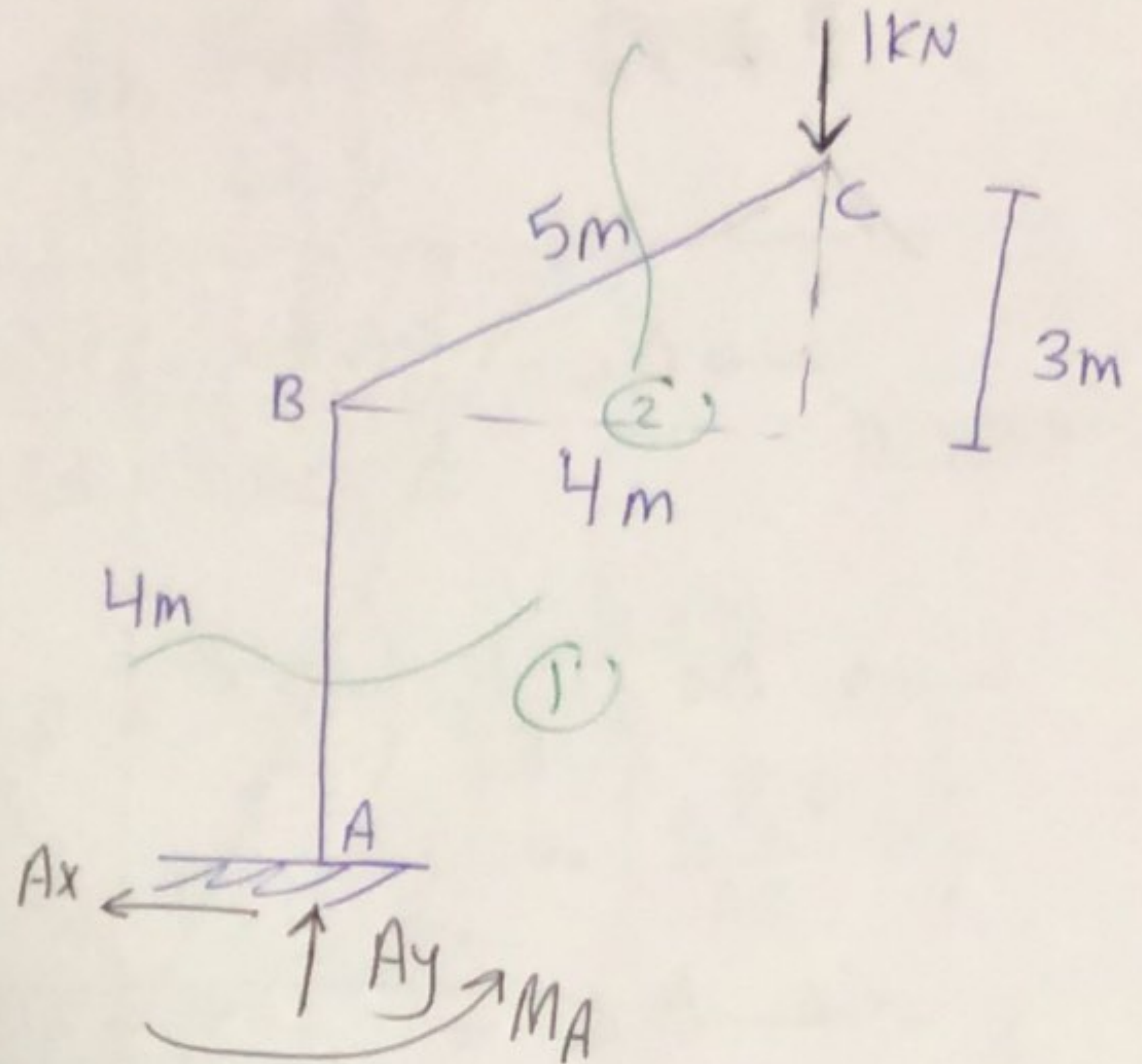
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- The Virtual:-

$$\sum F_y = 0 : A_y = 1 \text{ KN } \uparrow$$

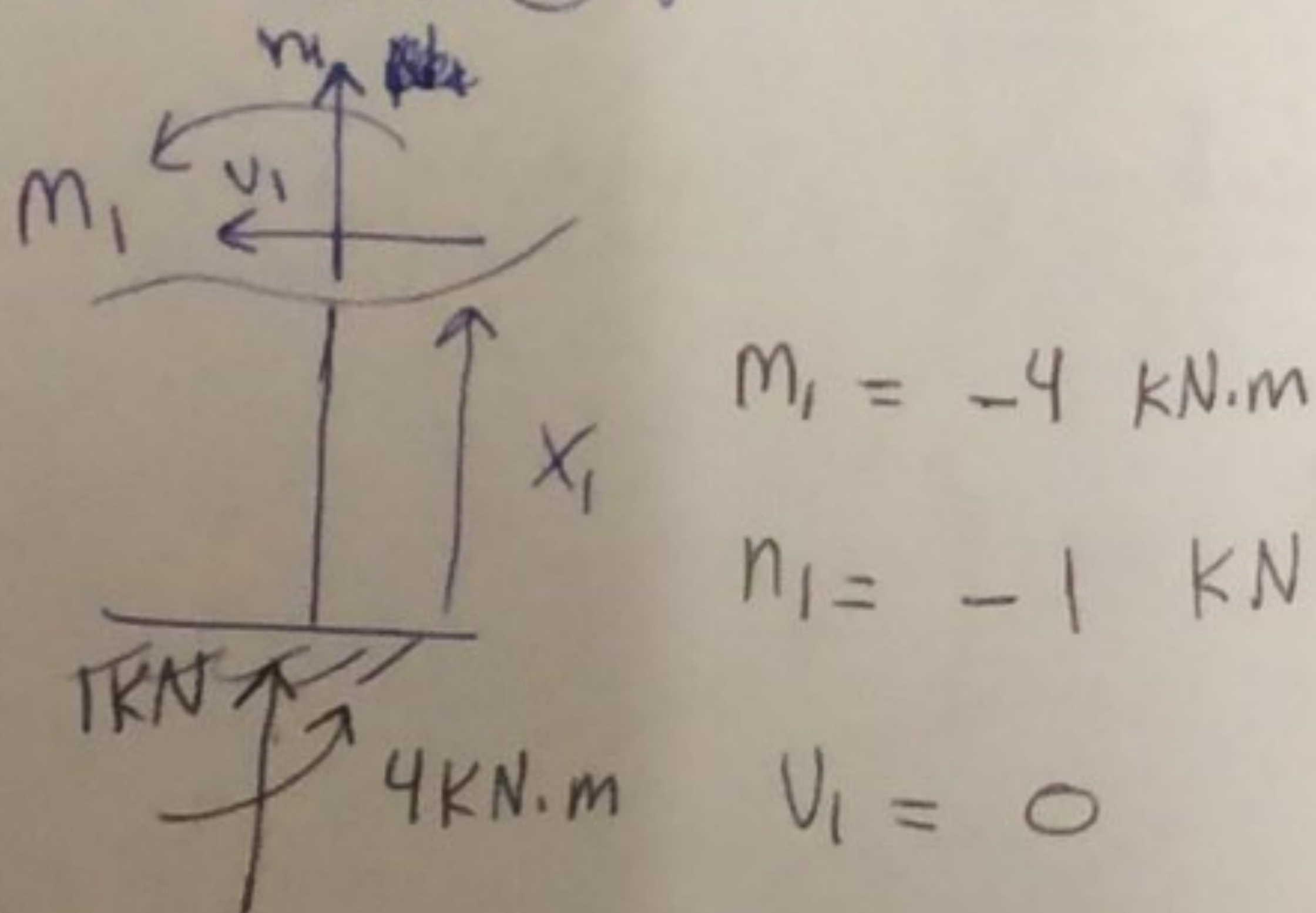
$$\sum M_A = 0 : M_A = 4 \text{ KN.m}$$

$$A_x = 0$$



For Virtual:-

Section ①:

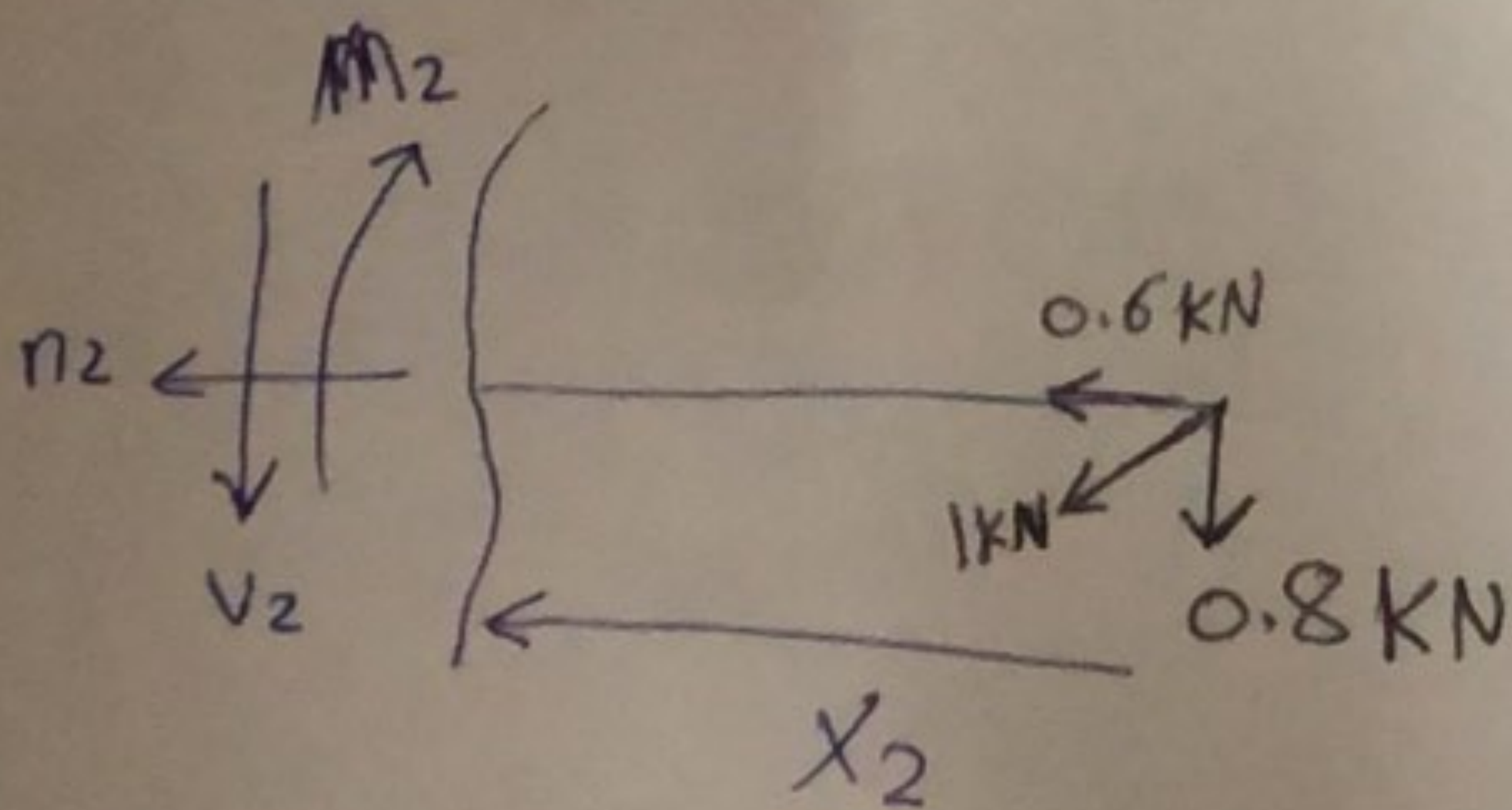


$$M_1 = -4 \text{ KN.m}$$

$$n_1 = -1 \text{ KN}$$

$$V_1 = 0$$

Section ②:



$$M_2 = -0.8 x_2 \quad (0 < x_2 < 5)$$

$$n_2 = -0.6 \text{ KN}$$

$$V_2 = -0.8 \text{ KN}$$

$1 * \Delta_c =$ Internal Virtual Work

Normal

Shear

Moment

$$\Delta_c = \left(\frac{n_1 N_1 L_{AB}}{AE} + \int_0^5 \frac{n_2 N_2}{AE} dx \right) + \left(\int_0^4 \frac{v_1 V_1 k}{AG} dx + \int_0^5 \frac{v_2 V_2 k}{AG} dx \right) + \left(\int_0^4 \frac{M_1 m_1}{EI} dx + \int_0^5 \frac{M_2 m_2}{EI} dx \right)$$

$0, (v_1 = 0)$

$$= \frac{-50(-1)(4)}{AE} + \int_0^5 \frac{6x_2(-0.6)}{AE} dx + \int_0^5 \frac{(-0.8)(-8x_2)(1.2)}{AG} dx + \int_0^4 \frac{(20x_1 - 2.5x_1^2 - 140)(-4)}{EI} dx + \int_0^5 \frac{-4x_2^2 \cdot -0.8x_2}{EI} dx$$

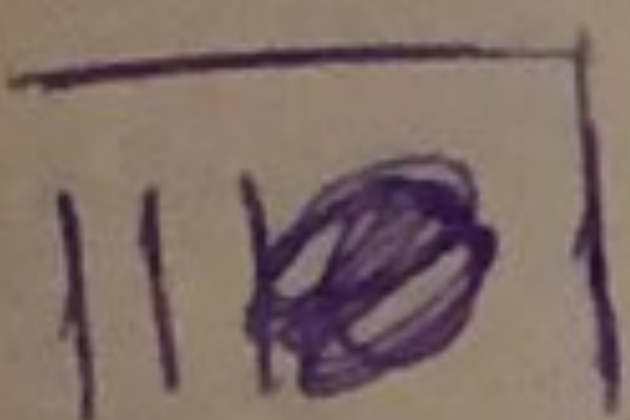
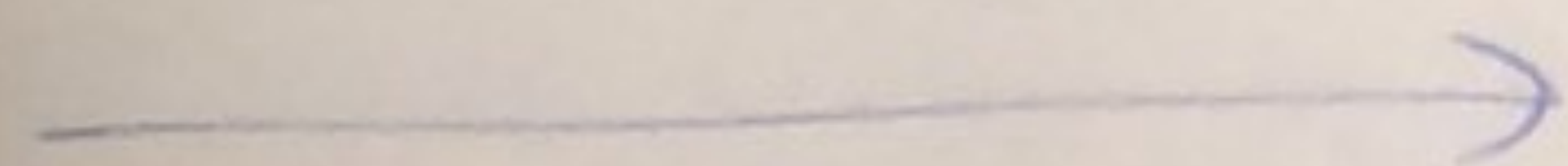
$$= \frac{200}{AE} + \frac{-45}{AE} + \frac{96}{AG} + \frac{-640 + 213.33 + 2240 + 500}{EI}$$

$$= \frac{155}{AE} + \frac{96}{AG} + \frac{2313.333}{EI}$$

Normal

Shear

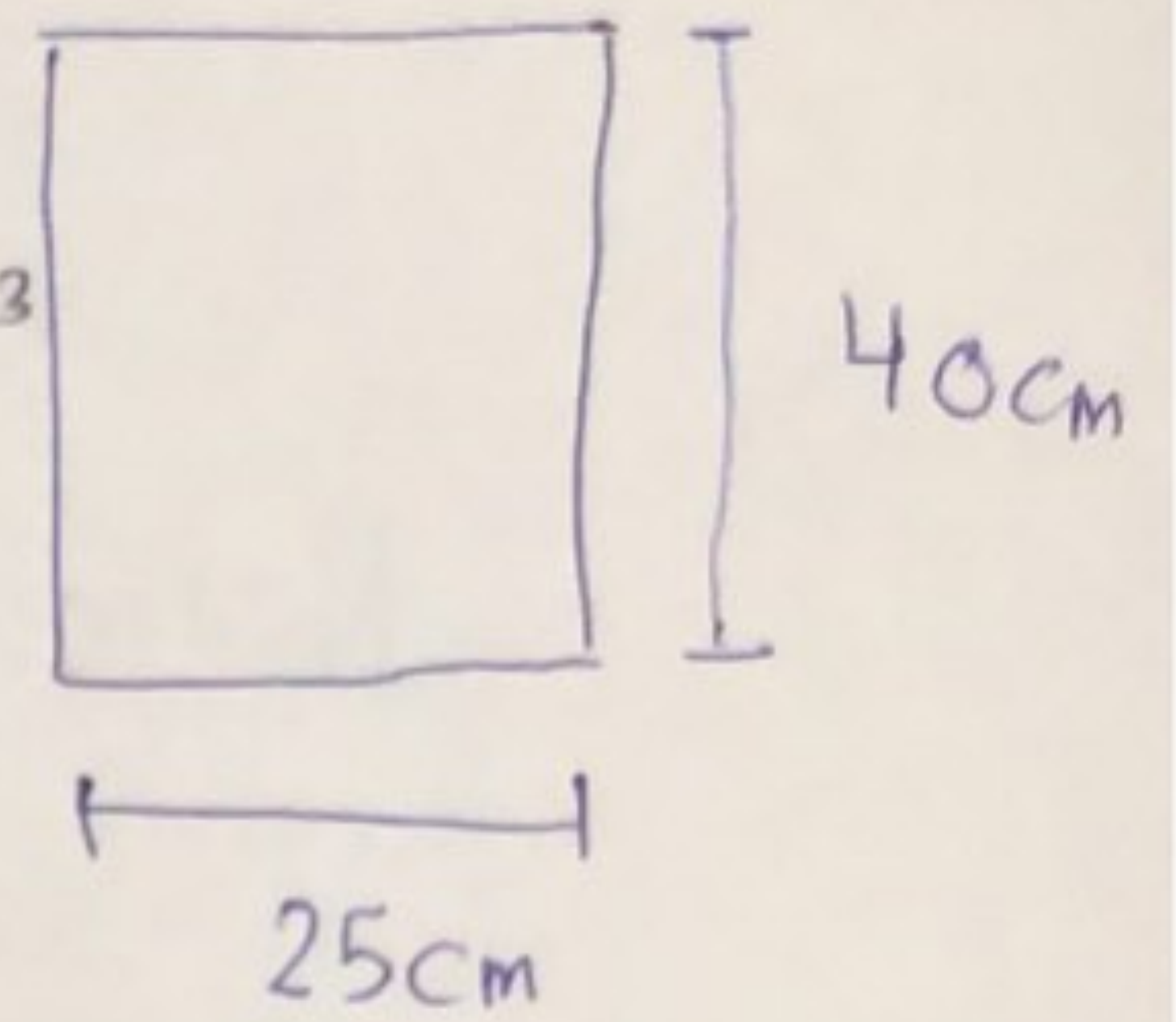
Moment



$$\Delta_C = \frac{155}{0.1 * 25 * 10^8} + \frac{96}{0.1 * 12 * 10^6} + \frac{2313.333}{25 * 10^6 * 1.333 * 10^{-3}}$$

$$= 62 * 10^{-6} + 80 * 10^{-6} + 0.06941$$

$$= 0.069552 \text{ m}$$



$$I = \frac{1}{12} (25) (40)^3$$

$$= 133333.333 \text{ cm}^4$$

$$= 1.333 * 10^{-3} \text{ m}^4$$

$$A = 0.1 \text{ m}^2$$

$$E = 25 \text{ GPa}$$

$$G = 12 \text{ GPa}$$